

# Chapter 13

## STRUCTURAL ANALYSIS AND EVALUATION

**NDOT STRUCTURES MANUAL**

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## Chapter 13

# STRUCTURAL ANALYSIS AND EVALUATION

Section 4 of the *LRFD Specifications* discusses the methods of structural analysis for the design and evaluation of bridge superstructures; analysis procedures for substructures are not specifically discussed in Section 4. Chapter 13 provides an elaboration on the provisions of LRFD Section 4 to discuss specific NDOT practices on structural analysis. [Chapters 17](#) and [18](#) provide provisions on structural analysis procedures for foundations and substructures (e.g., seismic).

### 13.1 LIVE-LOAD DISTRIBUTION

#### 13.1.1 General

Reference: LRFD Article 4.6.3.1

##### 13.1.1.1 Definition

Live-load distribution, for application of the *NDOT Structures Manual*, refers to the determination of the maximum number of loaded lanes that an individual girder of the superstructure will be expected to carry.

##### 13.1.1.2 Modeling Concrete Bridge Rails

The *LRFD Specifications* allows the structural contribution of any structurally continuous railing, barrier or median to be used to resist transient loads at the Service and Fatigue-and-Fracture limit states as a part of the cross section of the exterior girder. NDOT does not permit this allowance of structural contribution for new designs, but it may be considered in the evaluation or design for bridge rehabilitation if the contribution of the railing, barrier or median is significant.

#### 13.1.2 Approximate Methods

Reference: LRFD Article 4.6.2

##### 13.1.2.1 General

Traditionally, bridges have been analyzed using live-load distribution factors. These distribution factors result in a simple, approximate analysis of bridge superstructures. Live-load distribution factors uncouple the transverse and longitudinal distribution of force effects in the superstructure. Live-load force effects are assumed to be distributed transversely by proportioning the design lanes to individual girders through the application of distribution factors. The force effects are subsequently distributed longitudinally between the supports through the one-dimensional (1-D) structural analysis over the length of the girders.

Distribution factors reduce the necessity of modeling the entire bridge from a 2-D or 3-D analysis to a 1-D analysis of a girder. This 1-D, line-girder analysis is NDOT's preferred method of analysis, where suitable.

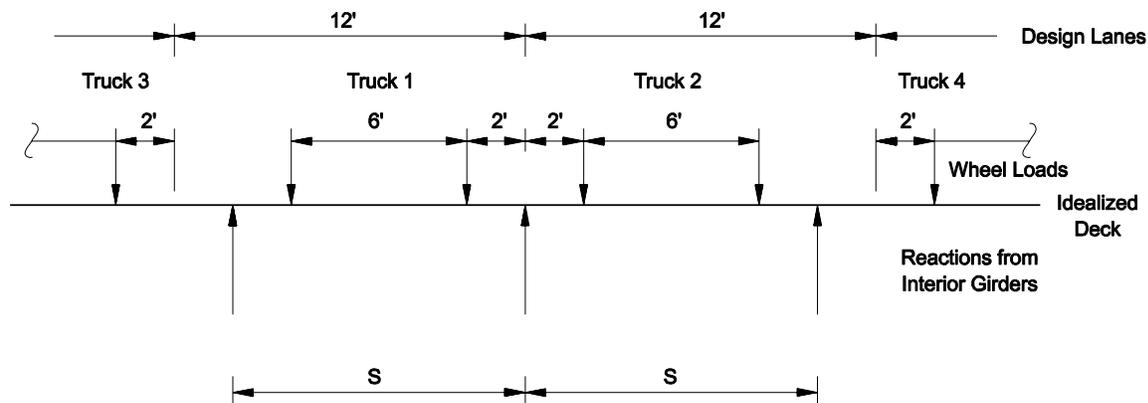
### 13.1.2.2 Simplified Analysis

Reference: LRFD Article 4.6.2.2

#### 13.1.2.2.1 General

LRFD Article 4.6.2.2.2 presents several common bridge superstructure types, with empirically derived equations for live-load distribution factors for each type. These more sophisticated distribution-factor equations are analytically superior to the former AASHTO *Standard Specifications* "S-over" factors that have been used for bridges with spans and girder spacings far beyond those for which they were originally intended. Each distribution factor provides a number of design lanes to be applied to a girder to evaluate the girder for moment or shear. The factors account for interaction among loads from multiple lanes.

The distribution factors represent the placement of design lanes to generate the extreme effect in a specific girder as illustrated in [Figure 13.1-A](#). The location of design lanes is not related to the location of striped lanes on the bridge. Summing all of the distribution factors for all girders produces a number of design lanes greater than the bridge can physically carry. This apparent overdesign occurs because each girder must be designed for the maximum load to which it could individually be subjected. Collectively, the individual load conditions producing the distribution factors cannot exist simultaneously on the bridge, yet each girder must be designed for its own worst case.



**DESIGN LANE AND TRUCK PLACEMENT PRODUCING THE WORST CASE FOR AN INDIVIDUAL INTERIOR GIRDER**

**Figure 13.1-A**

#### 13.1.2.2.2 *Limitations*

The tables of distribution-factor equations given in LRFD Article 4.6.2.2 include a column entitled “Range of Applicability.” The *LRFD Specifications* specifies that bridges with parameters falling outside the indicated ranges be designed using the refined analysis requirements of LRFD Article 4.6.3. In fact, these ranges of applicability do not necessarily represent limits of usefulness of the distribution-factor equations, but the ranges represent the range over which bridges were examined to develop the coefficients and exponents of the empirical equations. Other State DOTs have conducted parametric studies to study the use of these equations beyond these ranges for typical bridges in their States. These studies have demonstrated that the factors may be used beyond the range of parameters that were specifically studied. However, it is NDOT policy to require the approval of the Chief Structures Engineer before using the distribution-factor equations beyond the “Range of Applicability” without the use of a refined analysis. See [Section 13.2](#) for a discussion on refined analyses.

#### 13.1.2.2.3 *Skewed Bridges*

Simplified analyses using the specified distribution factors of LRFD Article 4.6.2.2 can be used for skewed bridges provided that adjustments are made.

The bending moment in the longitudinal direction in a skewed bridge is generally smaller than the bending moment in a rectilinear bridge of the same span. NDOT currently does not take advantage of the reduction in load distribution factors for moment in longitudinal girders on skewed supports.

Torsional moments exist about the longitudinal axis in skewed bridges due to gravity loads (both dead and live load). These moments increase the reactions and shear forces at the obtuse corners compared to the acute corners.

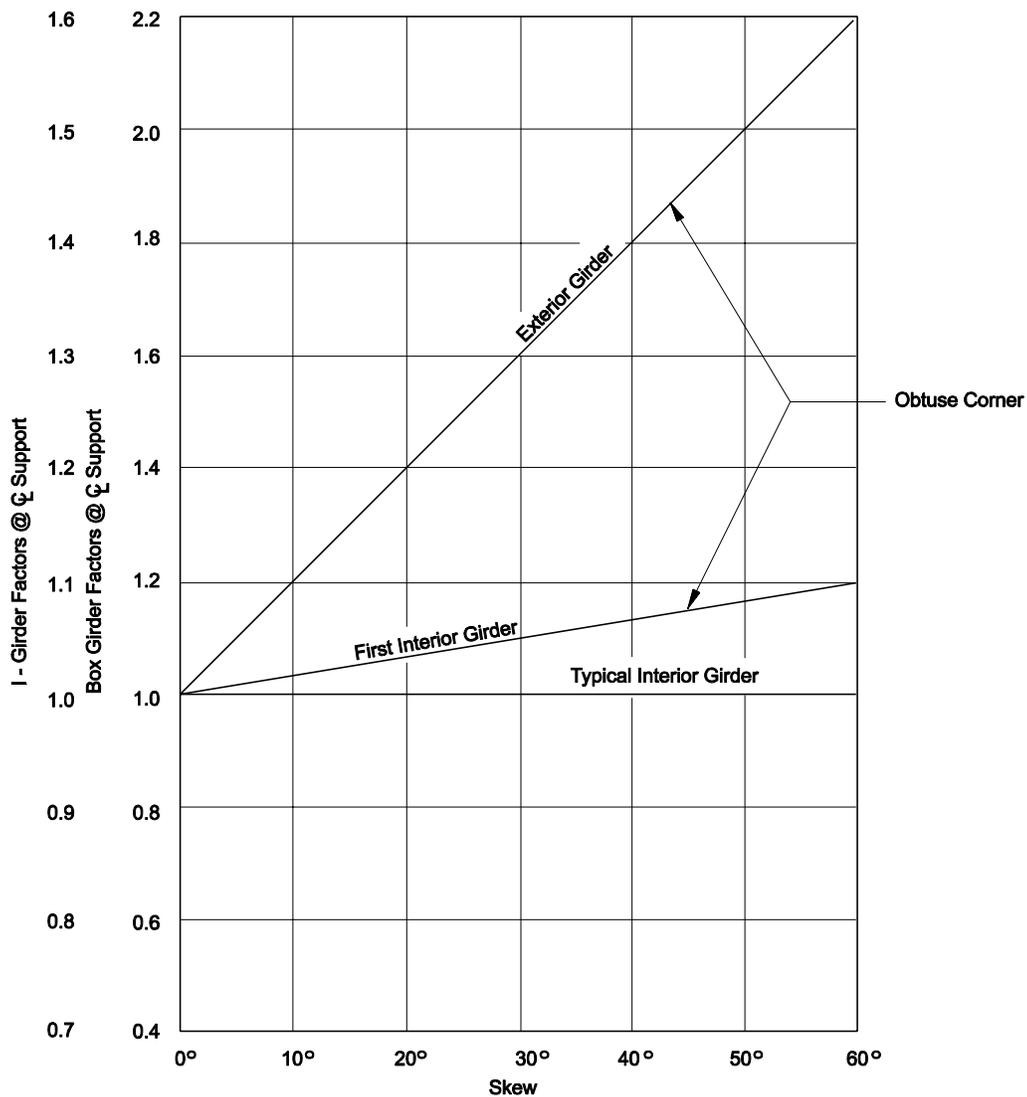
The potential exists for reactions to become very small or negative at acute corners, and should be avoided whenever possible during design. This can be achieved in post-tensioned bridges by the appropriate choice of the prestressing forces and the tendon profiles. The bridge designer should account for the higher reactions at the obtuse corners in the design of bearings and the supporting elements.

The skew correction factors for shear of LRFD Table 4.6.2.2.3c-1 shall be used to adjust the live load shears and reactions in skewed bridges. [Figure 13.1-B](#) shall be used to adjust the dead load shears and reactions. For shear design, the factors are assumed to vary linearly from the maximum value at the support to unity at midspan.

Curved bridges with supports skewed off of the radial direction by relatively large skew angles should be analyzed using a refined analysis; see [Section 13.2](#).

### 13.1.3 **Example**

The following presents an example of the live-load distribution factors for the approximate analysis of a cast-in-place, post-tensioned box girder.



DEAD LOAD SHEAR AND RESISTANCE FACTORS FOR SKEWED BRIDGES

Figure 13.1-B

\* \* \* \* \*

Given: Cross Section (see [Figure 13.1-C](#)). The span length = 160 ft.

Problem: Determine the live-load distribution factors for moment and shear.

Solution: Reference: LRFD Article 4.6.2.2

### **Distribution Factors for Moment**

Interior Girders: Reference: LRFD Table 4.6.2.2.2b-1

Two or more design lanes loaded:

$$g = \left( \frac{13}{N_c} \right)^{0.3} \left( \frac{S}{5.8} \right) \left( \frac{1}{L} \right)^{0.25}$$

Where:

$N_c$  = number of cells in a concrete box girder = 4  
 $S$  = spacing of girders or webs (ft) = 9.25 ft  
 $L$  = span of girders (ft) = 160 ft

$$g = \left( \frac{13}{4} \right)^{0.3} \left( \frac{9.25}{5.8} \right) \left( \frac{1}{160} \right)^{0.25} = 0.64$$

Whole-Width Design: Reference: LRFD Table 4.6.2.2.1

$$\begin{aligned} g_{\text{interior girder}} &= 0.64 \\ \text{No. of girders} &= 5 \\ g &= (5)(0.64) = 3.20 \end{aligned}$$

### **Distribution Factors for Shear**

Interior Girder: Reference: LRFD Table 4.6.2.2.3a-1

Two or more design lanes loaded:

$$g = \left( \frac{S}{7.3} \right)^{0.9} \left( \frac{d}{12.0L} \right)^{0.1}$$

Where:

$d$  = depth of girder (in) = 84 in

$$g = \left( \frac{9.25}{7.3} \right)^{0.9} \left( \frac{84}{12.0 \times 160} \right)^{0.1} = 0.90$$



Whole-Width Design: Reference: LRFD Table 4.6.2.2.1

$$g_{\text{interior girder}} = 0.90$$

$$\text{No. of girders} = 5$$

$$g = (5)(0.90) = 4.50$$

**Summary**

<b>Force Effect</b>	<b>Interior Girder</b>	<b>Whole-Width Bridge</b>
Moment	0.64	3.20
Shear	0.90	4.50

\* \* \* \* \*

## 13.2 REFINED ANALYSIS

Reference: LRFD Articles 4.6.2.2 and 4.6.3

### 13.2.1 General

Refined analyses include both 2-D and 3-D models (sometimes called grid and finite-element models, respectively). 2-D models are composed of elements lying in a single plane with the third dimension represented only through the stiffness properties of the elements. (The approximate methods of analysis of LRFD Article 4.6.2 employing distribution factors are essentially 1-D models where the only dimension used in the analysis is span length.) Typically, in a grid analysis, longitudinal elements represent the girders including any composite deck, and the transverse elements represent the deck. 3-D models are composed of elements in all three dimensions or of elements with three dimensions (such as brick elements). LRFD Article 4.6.3.3 provides general requirements for grid and finite-element analyses in terms of numbers of elements and aspect ratios.

### 13.2.2 2-D Analysis

#### 13.2.2.1 **Straight, Zero-Skew Bridges**

A 2-D analysis is only warranted for a straight, zero-skew bridge with the complicated geometry of non-standard girder framing such as an urban interchange bridge or a bridge with varying width.

#### 13.2.2.2 **Horizontally Curved Bridges**

The design of all superstructures must account for the effect of curvature where the components are constructed on horizontal curves. The magnitude of the effect of horizontal curvature is primarily a function of the curve radius, girder spacing, span length, diaphragm spacing and, to a lesser degree, web depth and flange proportions. The effect of curvature develops in two ways. First, the general tendency is for each girder to overturn, which has the effect of transferring both dead and live load from one girder to another transversely. The net result of this load transfer is that some girders carry more load and others carry less. The load transfer is carried through the diaphragms and the deck. The second effect of curvature is the concept of flange bending caused by torsion in curved components being almost totally resisted by horizontal shear in the flanges. The horizontal shear results in moments in the flanges. The stresses caused by these moments either add to or reduce the stresses from vertical bending. The torsion also causes warping of the girder webs.

Refined analysis methods, either grid or finite-element, shall be used for the analysis of horizontally curved bridges. LRFD Article 4.6.2.2.4 states that approximate analysis methods may be used for the analysis of curved bridges but then highlights the deficiencies of these analyses, specifically the V-load method for I-girders and the M/R method for boxes. Therefore, NDOT does not allow the use of these methods for curved bridges. The V-load method can be used for preliminary design purposes or as an order-of-magnitude checking tool.

### **13.2.2.3 Skewed Bridges**

Reference: LRFD Article 4.6.2.2.3c

A 2-D refined analysis may be warranted for skewed bridges with an angle of skew greater than 30°.

### **13.2.3 3-D Analysis**

A 3-D analysis, and its associated increase in costs, may not be warranted for the initial design of a bridge. For the analysis of complex structures or for the investigation of a problematic bridge (e.g., a bridge experiencing unexplained fatigue cracking), a 3-D analysis may be warranted.

## **13.3 SEISMIC ANALYSIS**

### **13.3.1 General**

The objective of seismic analysis is to assess the force and deformation demands and capacities on the structural system and its individual components. Equivalent static analysis (ESA) and linear elastic dynamic analysis (EDA) are the appropriate analytical tools for estimating the displacement demands for Ordinary Standard bridges. Inelastic static analysis (ISA) is the appropriate analytical tool to establish the displacement capacities for Ordinary Standard bridges.

### **13.3.2 Equivalent Static Analysis**

ESA can be used to estimate displacement demands for structures where a more sophisticated dynamic analysis will not provide additional insight into behavior. ESA is best suited for structures or individual frames with well-balanced spans and uniformly distributed stiffnesses where the response can be captured by a predominant translational mode of vibration. The seismic load shall be assumed as an equivalent static horizontal force applied to individual frames. The total applied force shall be equal to the product of the acceleration response spectrum (ARS) and the tributary weight. The horizontal force shall be applied at the vertical center of mass of the superstructure and distributed horizontally in proportion to the mass distribution.

### **13.3.3 Elastic Dynamic Analysis**

EDA shall be used to estimate the displacement demands for structures where ESA does not provide an adequate level of sophistication to estimate the dynamic behavior. A linear elastic multi-modal spectral analysis using the appropriate response spectrum shall be performed. The number of degrees of freedom and the number of modes considered in the analysis shall be sufficient to capture at least 90% mass participation in the longitudinal and transverse directions. A minimum of three elements per column and four elements per span shall be used in the linear elastic model.

EDA, based on design spectral accelerations, will likely produce stresses in some elements that exceed their elastic limit. The presence of such stresses indicates nonlinear behavior. The bridge designer should recognize that forces generated by linear elastic analysis could vary considerably from the actual force demands on the structure. Sources of nonlinear response that are not captured by EDA include the effects of the surrounding soil, yielding of structural components, opening and closing of expansion joints, and nonlinear restrainer and abutment behavior. EDA modal results shall be combined using the complete quadratic combination (CQC) method.

Typically, the entire bridge is modeled. For longer structures, the bridge designer should model a boundary frame (or abutment, where appropriate) at each end of the frame under investigation as a minimum.

### **13.3.4 Inelastic Static Analysis**

ISA, commonly referred to as “push-over” analysis, shall be used to determine the reliable displacement capacities of a structure or frame as it reaches its limit of structural stability. ISA shall be performed using expected material properties of modeled members. ISA is an

incremental linear analysis, which captures the overall nonlinear behavior of the elements, including soil effects, by pushing them laterally to initiate plastic action. Each increment pushes the frame laterally, through all possible stages, until the potential collapse mechanism is achieved. Because the analytical model accounts for the redistribution of internal actions as components respond inelastically, ISA is expected to provide a more realistic measure of behavior than can be obtained from elastic analysis procedures.

Structural system or global analysis is required when it is necessary to capture the response of the entire bridge system. Bridge systems with irregular geometry (especially horizontally curved bridges and skewed bridges, multiple transverse expansion joints, massive substructure components, and foundations supported by soft soil) can exhibit dynamic response characteristics that are not necessarily obvious and may not be captured in a separate subsystem analysis.

The two-dimensional plane frame “push-over” analysis of a bent or frame can be simplified to a column model (fixed-fixed or fixed-pinned), if it does not cause a significant loss in accuracy in estimating the displacement demands or the displacement capacities. The effect of overturning on the column axial load and associated member capacities must be considered in the simplified model. The simplified analytical technique for calculating frame capacity is only permitted if either Equations 13.3-1 and 13.3-2 or 13.3-3 and 13.3-4 below are satisfied. . Equations 13.3-1 and 13.3-3 apply to any two columns within a bent and any two bents within a frame. Equations 13.3-2 and 13.3-4 apply to *adjacent* columns within a bent and *adjacent* bents within a frame.

For constant-width frames:

$$\frac{k_i^e}{k_j^e} \geq 0.5 \quad (\text{Equation 13.3-1})$$

$$\frac{k_i^e}{k_j^e} \geq 0.75 \quad (\text{Equation 13.3-2})$$

For variable-width frames:

$$\frac{k_i^e / m_i}{k_j^e / m_j} \geq 0.5 \quad (\text{Equation 13.3-3})$$

$$\frac{k_i^e / m_i}{k_j^e / m_j} \geq 0.75 \quad (\text{Equation 13.3-4})$$

Where:

- $k_i^e$  = the smaller effective bent or column stiffness
- $k_j^e$  = the larger effective bent or column stiffness
- $m_i$  = tributary mass of column or bent i
- $m_j$  = tributary mass of column or bent j

In addition, the ratio of fundamental periods of vibration for adjacent frames in the longitudinal and transverse direction shall satisfy Equation 13.3-5:

$$\frac{T_i}{T_j} \geq 0.7 \quad \text{(Equation 13.3-5)}$$

Where:

$T_i$  = natural period of the less flexible frame  
 $T_j$  = natural period of the more flexible frame

### 13.4 INELASTIC REDISTRIBUTION OF GRAVITATIONAL FORCE EFFECTS

Reference: LRFD Appendices A6 and B6

The *LRFD Specifications* presents simplified approaches to inelastic redistributions of moments in girder bridges. LRFD Article 5.7.3.5 provides a simple multiplier for negative-moment redistribution based upon ductility of a reinforced concrete section. LRFD Articles 6.10 and 6.11 present a simplified approach to inelastic redistribution of moments in steel girder bridges. The simplified approach allows the moment in a section to approach 1.3 times the moment at first yield, acknowledging the inherent ability of positive moments to inelastically redistribute to negative-moment steel sections regardless of the compactness of the negative-moment section.

NDOT prohibits the use of the LRFD Appendices to Section 6, which include more rigorous inelastic procedures for steel girders. LRFD Appendix A6 specifies a more rigorous and thus more extensive redistribution of positive moments to compact negative-moment sections. LRFD Appendix B6 gives similar provisions for the redistribution of moments at compact negative-moment sections.

